Rotorcraft Weight and Center of Gravity Estimation

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Abstract

An accurate, automated assessment of helicopters’ gross weight (GW) and center-of-gravity (CG) is critical for the determination of aircraft fatigue and life estimates since GW/CG greatly affect static and dynamic characteristics of helicopters. Current methods of GW and CG data gathering requires not only meticulous bookkeeping but it also makes several assumptions about fuel burn rate if fuel quantity instrumentation is not available. In addition, if cargos are released/picked-up during a flight, a log of loads and time instants is required to ensure aircraft GW and CG are known at all times during the flight. This paper reviews current state of the art for the GW and CG estimations’, summarizing different techniques and underlining their advantages and limitations. Preliminary results are presented in order to underline the capabilities of the proposed techniques. Then, the general framework of an innovative, hybrid approach that combines Artificial Neural Networks (ANN) and Kalman Filter (KF) techniques is introduced.

Keywords: Helicopters gross weight and center of gravity, Helicopter system, HUMS data, Kalman Filter, Artificial Neural Network.

Introduction

The renewed interest in improving helicopter rotorcraft GW and CG estimates, especially in situations where sudden weight changes occur due to loads that are picked up or dropped off during flight motivates the current research. An accurate, automated assessment of helicopter GW and CG is critical for the determination of aircraft fatigue and life estimates since GW/CG greatly affect static and dynamic characteristics of helicopters. Therefore GW and CG of a helicopter are valuable information in calculating reliable loads and remaining fatigue life. These in turn will assist the condition based maintenance systems used to enhance safety and reduce the operating cost of helicopters. An automated system for GW and CG will improve aircraft structural life estimation and performance characteristics, will relieve pilot’s burden of logging data, and will also improve situational awareness. To capture GW and CG changes continuously throughout the flight, advanced methods are required as conventional methods are not sufficient and prone to errors.

As the problem of estimating GW and CG is very complex, a combination of physics based (deterministic) models and data driven (stochastic) models should be used. The deterministic models for which the outcomes are precisely determined through known relationships among states and events without any room for random variation have the following limitations: (1) no mathematical system model is perfect; (2) the helicopter is a dynamic system, driven not only by control inputs, but also by disturbances which cannot be controlled or modeled deterministically; (3) sensors, such as Health and Usage
Monitoring Systems (HUMS) do not provide perfect and complete data about the helicopter. Because of these limitations, stochastic models which define ranges of values for variables should also be used. Therefore, it is believed that a good model for estimating helicopters’ GW and CG should include two knowledge sources: expert (physics-based, deterministic) and learning from examples (data-driven, stochastic).

This paper is structured as follows: after this short introduction, a review of general models used to calculate the helicopters’ GW and CG is presented, followed by a general introduction to ANN based models which are illustrated using an example. Then an introduction to KF based models and a second example are presented. In the end an innovative, hybrid approach that combines ANN and KF techniques is detailed and future work is given.

**General Models**

Ref. 1 provides a simple model to estimate the helicopters GW, based on the ground/takeoff weight and updates approximated during the flight. These updates are obtained based on the change in the fuel quantity by a direct measurement or estimated based on a fuel burn rate. The model limitations are associated with possible inaccuracy in the take-off weight and difficulties associated with fuel consumption models. Also a detailed bookkeeping need to be used for sudden weight changes during flight and this model does not provide the CG.

Another straightforward model is illustrated by Ref. 2. In this case, the hover performance chart found in the operator’s user manual and flight parameters such as engine torque, hover height, pressure altitude, and ambient temperature are used to estimate GW. However the model gives only a weight range (low, medium, and high) and only when the helicopter is in hover conditions. Another drawback of the models is related with the fact that it relies on a performance chart, which may be inaccurate. Also, as the previous model, the CG cannot be estimated.

A combination of flight parameters and an analytical model is employed in Ref. 3 to estimate GW. The inertial sensor data, air data system measurements and the corrected momentum theory are applied to estimate shaft horsepower, and then the difference between the actual and estimated shaft horsepower is used to adjust the estimate of gross weight until the estimated power level tracks the actual power level. Although a simple and straightforward solution, the model estimates only GW and is not intended for real-time usage.

**Neural Network – Learning from Examples**

**Neural Network – Overview**

As schematically shown in Fig. 1, NN is an adaptive system that changes its structure based on external (target) and internal (output) information that flows through the network [4, 5]. This modular structure provides a multitude of efficient training algorithms for computing the NN parameters so as to best fit the target set.
Fig. 2 illustrates a feed forward, back propagation NN with few layers of neurons: the input layer, two hidden layers and the output layer. This type of NN is the most common NN used for this problem and it is called feed forward, back propagation because the signal propagates forward whereas the learning propagates backward. The feed forward, back propagation NN is as a directed graph with: (1) a state variable associated with each node; (2) a weight associated with each link between two nodes; (3) a bias associated with each node, and (4) a transfer function for each node which determines the state of the node as a function of its bias, the weights of its incoming links, and the states of the nodes connected to it by those links.

Fig. 1: Simplified view of the NN process

Fig. 2: Simplified view of a feed forward, back propagation NN

First step of NN technique consists in selecting the input data and defining the training and validation data sets along with network architecture. The training set is used to define the model internal parameters whereas the validation data set is used to assess model performance. The network architecture is defined by selecting the type and the geometry. During the second step named learning/training, the prediction error is propagated through the network in some manner and used to adjust the network weights such that the prediction error is minimized. The last step consists in testing the network using the validation data set.
Neural Network-based Models

NN is among the relatively straightforward methods also proposed for GW and CG estimation [6-9]. Same steps are applied in all the cases, the differences being the aircraft considered (SH-60B in Ref. 6, V-22 for in Refs. 7 and 8), the source of input parameters (real flight data in Ref. 6 and 7 and simulated flight data in Ref. 8), the NN architecture and type (radial basis or back-propagation) and the selection of the input parameters (4 parameters in Ref. 6 and 13 parameters in Ref. 7 and 8). One of the advantages of this method is the freedom to identify and quantify the most significant parameters for prediction. Ref. 6 considers only 4 HUMS recorded data: engine torque, longitudinal stick position, altitude, collective stick position whereas Refs. 7 and 8 considers 13 HUMS recorded data: left rotor torque, right rotor torque, left rotor longitudinal cyclic control, right rotor longitudinal cyclic control, left rotor lateral cyclic control, right rotor lateral cyclic control, nacelle angle, pedal position, pitch attitude, roll attitude, radar altitude, density altitude, normal load factor. In all the cases, the results of the NN approach are validated by comparison with the measurements and the root mean square (RMS) error is relatively small.

Nevertheless, NN-based methods have several limitations: it is applied to a specific flight condition (e.g. high speed flight, hover or level flight); the estimation accuracy is dependent on the available and accurate data; a multi-point moving average needs to be used to smooth the input data; the accuracy depends upon network being rigorously trained, which is invariably a time-consuming and laborious process; the aircraft time degradations cannot be taken into consideration.

Although applied to a fixed wing aircraft, Ref. 9 gives an innovative model that combines NN with an analytical model. Using measured Mach number, angle of attack, elevator deflection, and dynamic pressure as inputs and lift, drag and aerodynamic pitch moment coefficient as outputs of a NN, the model calculates GW and CG based on a simple aerodynamics model. The advantage of the model is that it estimates a posteriori mass center using the lift and pitching moment functions. The limitation of this model, however, are that it addresses steady/ trimmed level flight only and it is based on a simplistic analytical model.

Neural Network-based Model – an Example

In order to illustrate these concepts, this section presents a simplified example of a NN-based technique. The data is generated using flight dynamics software, and a feed-forward, back-propagation NN is built to train and test the data. The true and predicted values of the mass are compared in order to evaluate the model capabilities. In this example the mass and not GW is calculated and plotted, but GW can be easily computed based on the mass values.

The most important aspect of the NN method is the availability and accuracy of the data for both training/learning and testing/validating the NN. As real flight data is not available at the current moment, a software is used to generate the needed data. Commercial available, real-time helicopter flight dynamics software, RotorLib FDM (flight dynamics model) is used to find trimmed states for forward flight conditions [10]. As illustrated in Fig. 3, RotorLib FDM has a component based architecture where several mathematical
models interact with each other. These are: Momentum theory based rotor model; Fuselage model; Stabilizer model; Rigid-body dynamics based landing gear model; Control system models (e.g. stability augmentation systems); Instrument models; Earth model. The forces and moments from the different elements of a helicopter, such as main rotor, tail rotor, fuselage and empennage are computed and the equations of the translational and rotational motions of a helicopter assumed to be a rigid body are utilized. The helicopter has six degree of freedom in its motion, it has eighteen state variables (position, velocity, angular velocity, Euler’s angles, longitudinal and lateral 1st harmonic flapping of both main and tail rotor) and four control input parameters (collective, roll-pitch lateral and longitudinal cyclic and pedal).

![Diagram showing the interaction of different models in a helicopter system](image)

**Fig. 3: RotorLib FDM – Rigid Body Dynamics overview based on a typical utility helicopter configuration [10].**

Typical UH-60 helicopter data, gathered from various sources (e.g. [11, 12]) are used as inputs in RotorLib FDM. As illustrated in Fig. 4, three input parameters are varied for a total of 900 flight conditions: altitude varies from 5 to 2505 m; speed varies from 5 to 75 m/s whereas mass varies from 4500 to 9000 kg. In all the cases, the trim solver is run in order to find trimmed states for the level flight. Fig. 5 shows 85% of the data i.e. 765 flight conditions (drawn as circles) randomly selected for training, whereas the rest is used for validating.

The helicopter dynamic model has 78 scalar outputs and many of them are relevant for the mass estimation. However, the main goal of this example is to illustrate the NN application to such a problem, and not necessarily to get an accurate mass estimation. For this reason, only the power which is defined as a sum of the main rotor and the tail rotor powers is calculated and used for mass estimation. Note that as the number of neurons in the input layer increases, the error in mass estimation will decrease. Fig. 6 shows the calculated power for all 900 flight conditions. The data is plotted using dot symbol whereas the training data set (representing 85% of the data) is plotted using circle symbol.
Fig. 4: Flight condition matrix used for NN: 900 points for various speed, altitude and mass.

Fig. 5: Flight condition matrix used for NN: out of 900 data (drawn as points), 765 (drawn as circles) are used for training

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Fig. 6: Calculated power for all 900 flight conditions (drawn as points) and randomly selected data training set (drawn as circles)

Fig. 7 shows the feed-forward, back-propagation NN architecture. The input layer has three parameters: speed, altitude and power. Two hidden layers, one with six neurons and one with three neurons are used and the output layer consists of only one neuron which defines the mass/weight. The weights associated with each link between two nodes are shown also: the thicknesses of the links are proportionally with their magnitudes and negative weights are shown in red whereas the positive ones are shown in green. A software called Easy NN-plus [13] is used to train and test the data.

The true and predicted values of the mass are compared in Fig. 8 in order to evaluate the model capabilities: Fig. 8(a) presents a comparison for the training set and shows that the average training error is below 1% whereas the maximum training error is approximately 7%. Fig. 8(b) displays a comparison for the validation set and shows that 95% of the validating examples are within 4% of the desired outputs. As stated above, this is just a simplistic example used to illustrate the applicability of the NN technique for mass estimation. It is expected that these errors to decrease if the number of inputs in the NN increases.
Fig. 7: Architecture of the feed-forward, back-propagation NN used: 3 neurons in the input layer, two hidden layers, and one neuron in the output layer.

Fig. 8: True values vs. predicted values: (a) training set (the average training error is below 1%); (b) validating set (95% of the validating examples are within 4% of the desired outputs)

Kalman Filter – Expert Knowledge

Kalman Filter – Overview

KF is a recursive technique that estimates the state of a dynamic system from a series of incomplete and/or noisy measurements [14]. It is a sensor fusion algorithm because it uses a system's dynamics model (i.e. physical laws of motion) and measurements from sensors.
in order to form an estimate of the system's varying quantities (its state) that is better than the estimate obtained by using the measurement or the analytical model alone. The two sources of uncertainties are the noisy sensor data and the approximations in the equations that describe how a system changes.

In order to illustrate KF concepts, a discrete time linear system schematically presented in Fig. 9(a) is considered. This system represented in a state variable format is written as:

$$ x_j = ax_{j-1} + bu_j + w_j $$

where $x_j$ is the state at current time instant $j$, $a, b$ are constants, $u_j$ is the input and $w_j$ is the process noise. It is assumed that the state $x_j$ is not directly measured but instead another variable is measured (Fig.9 (b)):

$$ z_j = hx_j + v_j $$

where $h$ is the gain and $v_j$ is the measurement noise at the current time instant $j$.

The main goal of the KF technique is to filter $z$ so as to estimate the state $x$ while minimizing the effects of $w$ and $v$. This is accomplished in two steps, schematically shown in Fig.9 (c):

1. Time Update (“Predict”)
   a. Project the state ahead: $\hat{x}_{j-} = a\hat{x}_{j-1} + bu_{j-1}$
   b. Project the error covariance ahead: $P_{j-} = aP_{j-1}a + Q$

2. Measurement Update (“Correct”)
   a. Compute the Kalman gain: $K_j = P_{j-}h(hP_{j-}h + R)^{-1}$
   b. Update estimate with measurement $z$: $\hat{x}_j = \hat{x}_{j-} + K_j(z_j - h\hat{x}_{j-})$
   c. Update the error covariance: $P_j = (I - K_jh)P_{j-}$

KF is relatively popular because it gives good results in practice due to its optimality and structure, it has a convenient form for online real time processing and it is easy to formulate and implement given a basic understanding. For all these reasons, it is a promising technique for the current problem.
Fig. 9: (a) Discrete time linear system (where $T$ is a time delay); (b) Discrete time linear system with measurements; (c) Schematic of the KF technique underlying the two required steps: predictor and corrector.

Kalman Filter-based Model

Ref. 15 presents another technique for GW and CG estimation, based on the helicopter dynamics model which is solved by an extended KF technique. In this case, the model represents an idealization of the real rotorcraft dynamics with a state vector which consists of weight and balance states as well as rigid vehicle states. The advantage of the model is that it can be used in hover and forward flight as well as situations where loads are dropped and picked up in flight. On the other hand, further work is needed in order to investigate the details of implementation challenges, for example estimation of the total forces and moments of the rotorcraft under various flight conditions.

Kalman Filter-based Model – an Example

In general a helicopter dynamics model can be represented by the following non-linear differential system:

$$\frac{dx}{dt} = f(x, u)$$  \hspace{1cm} (8)

$$y = g(x, u)$$  \hspace{1cm} (9)

where $x$ is the state vector, $y$ is the output vector whereas $u$ is the control input. Both $f$ and $g$ functions are general nonlinear functions. In order to apply the extended Kalman filter, Eqn. (8) needs to be linearized as follows:

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\[
\frac{dx}{dt} = Ax + Bu
\]  
(10)

For the current problem matrices \( A \) and \( B \) are not constants but functions of the unknown GW (or mass, \( m \)). For this reason, Eqn. (10) has to be written as:

\[
\frac{dx}{dt} = A(m)x + B(m)
\]  
(11)

Equation (11) needs to be linearized about a fixed point \((x_0, u_0, m_0)\) using a Taylor’s series expansion:

\[
\frac{dx}{dt} = A(m_0)x + B(m_0)u + M_1 m - M_1 m_0
\]  
(12)

where the matrix \( M_1 \) is defined as:

\[
M_1 = \frac{\partial A}{\partial m}(x_0, u_0, m_0)x_0 + \frac{\partial B}{\partial m}(x_0, u_0, m_0)u_0
= A(x_0, u_0, m_0 + \Delta m) - A(x_0, u_0, m_0)x_0
+ \frac{\Delta m}{\Delta m} B(x_0, u_0, m_0 + \Delta m) - B(x_0, u_0, m_0)u_0
\]  
(13)

Then Eqn. (12) can be written as:

\[
\begin{pmatrix}
\dot{x} \\
\dot{m}
\end{pmatrix} = 
\begin{bmatrix}
A(m_0) & M_1 \\
0 & 0
\end{bmatrix}
\begin{pmatrix}
x \\
m
\end{pmatrix} + 
\begin{bmatrix}
B(m_0) \\
0
\end{bmatrix}u + 
\begin{bmatrix}
-M_1 \\
0
\end{bmatrix}m_0
\]  
(14)

or in a more compact form:

\[\dot{x}_\tau = A_\tau x_\tau + B_\tau u + M_\tau m_0\]  
(15)

where \( x_\tau = [x \quad m]^T \) and matrices \( A_\tau, B_\tau, M_\tau \) are defined as:

\[
A_\tau = \begin{bmatrix}
A(m_0) & M_1 \\
0 & 0
\end{bmatrix}, \quad B_\tau = \begin{bmatrix}
B(m_0) \\
0
\end{bmatrix}, \quad M_\tau = \begin{bmatrix}
-M_1 \\
0
\end{bmatrix}
\]  
(16)

An extended Kalman filter is applied for the following equation:

\[\dot{x}_\tau = A_\tau x_\tau + B_\tau u\]  
(17)

whereas \( M_\tau m_0 \) term will be included into the error term.
In order to demonstrate these concepts the above described software, RotorLib FDM is employed. Using generic parameters for an UH-60 helicopter, the software is used to collect data for the flight shown in Fig. 10. Fig. 10(a) presents position whereas Fig. 10(b) presents velocity of the helicopter in the inertial (world) coordinate system. The initial condition is represented by hover at 1600 m and then the helicopter is flown for 50 s. It is assumed that in this short time interval the gross mass is constant, \( m = 725 \text{kg} \).

During the flight, matrices \( A \) and \( B \) are collected, and matrices \( A_T \) and \( B_T \) are computed. Then an EKF is applied for Eqn. (17) and the result is plotted in Fig. 11 which shows the mass: dotted line shows the constant true mass whereas the triangle symbols shows the EKF estimate. In the absence of real flight data, both the state (continuous line) and the measurement (circle symbol) include random generated errors. These errors need to be introduced according to term \( w_j \) of Eqn. (1) and \( v_j \) of Eqn. (2) and they represent uncertainties of the process and measurements. It can be noted from Fig. 11 that the maximum error of the mass estimated by the EKF is less than 1%.

Future work will include similar examples for the general case when the mass is variable and also a very similar example for calculating the CG.

![Fig. 10: (a) Position in the inertial frame (unit is m); (b) Velocity in the inertial frame (unit is m/s).](image)
Future Work: Hybrid NN/KF-based Model

As described above both NN-based and KF-based techniques for estimating GW and CG have strengths and limitations. A KF approach provides accurate state estimation in the presence of noisy, biased, or missing measurements due to fusion between the sensor data and physical system model. At the same time KF is based on an analytical model which represents an idealization of the rotorcraft dynamics, therefore total forces and moments need to be known in order to solve the differential system and the influence of modeling error can be large.

On the other hand NN-based approach is simple and straightforward, it has the freedom to identify and quantify the most significant parameters for prediction and its structure provides a multitude of efficient training algorithms for computing the parameters so as to best fit the training set. But then again using NN models alone will not provide a CG calculation because no data is available; the estimation accuracy will depend on availability and accuracy of the data; and the network will need rigorous training, which is invariably a time-consuming and laborious process. Also in order to include the rotorcrafts’ degradation of performance over time, the NN needs to be retrained.

It is believed that combining both methods will overcome each technique limitations and will take advantage of each method’s strengths. The future work will investigate several hybrid concepts with a final goal to develop a new model that will estimate GW/CG with 1% - 2% accuracy. For example, Fig. 12 illustrates a possible combination of the two models. In this case NN is the main process, whereas KF is a source used to generate data for NN training. These model-based data in combination with real flight data (e.g. Engine Torque, Altitude, Airspeed, Yaw rate, Sideslip, Pitch and Roll Attitude, etc.) will be used in a NN in order to compute the GW and CG for a helicopter. The model needs further development in order to understand its advantages and limitations.
Conclusions

This paper reviews the current state-of-the-art for helicopters GW and CG estimation, summarizing different techniques and underlining their advantages and limitations. Two simple examples are presented in order to underline the capabilities of the current techniques of NN and KF in calculating the GW.

Data regression-based models as proposed in this paper will be beneficial for diagnostic/prognostic purposes as well as supplementing/complementing in-flight GW and CG data gathering even if there are hardware-based solutions for obtaining near real-time GW and CG because of possible gaps in sensors data.

Future work includes development of an innovative, hybrid approach that will combine powerful estimation capabilities of the KF scheme with the strong learning capabilities of the NN. Linking NN and KF solutions will provide original ways to solve more general rotorcraft problems in this domain, such as parameter identification and regime recognition.

References


13. EasyNN-plus help, the user interface manual, Copyright © 2002 – 2010 Neural Planner Software Ltd.
